Coupling Matrix Extraction for Cascaded Triplet (CT) Topology

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Abstract - Each triplet has the property of realizing a single real frequency transmission zero. Asymmetric filter responses can be realized by a cascade of triplet sections. This paper describes a Newton-Raphson method for rapid solution of the equations, which produces the desired coupling matrix for a CT topology.

I. INTRODUCTION

The synthesis procedure developed by Atia and Williams [1]-[4] for symmetrical filter responses and then revised by Cameron [5] for an asymmetric filter response produce a multiple coupled generic coupling matrix for given scattering parameters S_{21} and S_{11}. Then methods based on similarity transformations have been used to reduce the matrix to a realizable form [5], [6]. However, an exact sequence of similarity transformations to obtain the cascaded triplet topology is yet to be found. A different approach to this problem is presented in this paper.

In the synthesis, a multiple coupled generic matrix results from the orthonormalization procedure, which does not take the final topology into account. Our approach is to perform this orthonormalization procedure, while satisfying the network topology. This involves solving a system of non-linear equations, which is done using the Newton-Raphson method. The paper demonstrates how this procedure can be applied to synthesize cascaded triplets for filter orders 5, 6 and 7. The transmission coefficient calculated from the resultant coupling matrix agrees with great accuracy to the synthesized transmission coefficient.

The rest of the paper is arranged as follows. Section II describes the background and our approach to the problem. Then section III shows how it can be applied to synthesize a CT topology, followed by results in Section IV to validate the presented procedure. Finally, conclusions are given.

II. COUPLING MATRIX GENERATION USING THE NEWTON-RAPHSON METHOD FOR CASCADED TRIPLETS

The full coupling matrix can be defined as [5]

$$M = -T \Lambda T^t \quad (1)$$

where T is an orthonormal matrix and Λ is a diagonal matrix. All matrices are order N, where N is the filter order. The synthesis procedure given in [5] extracts the first and the last rows of T, the complete diagonal matrix Λ, and the terminal resistances R_1 and R_N for a given transmission coefficient (S_{21}) and reflection coefficient (S_{11}). The remaining rows of T could be found using the Gram-Schmidt procedure. Since this orthonormalization procedure is independent to the network topology, the resultant coupling matrix contains all possible couplings.

Our approach is to find the remaining N-2 orthonormal row vectors of T, which also satisfy the required network topology. It is possible to write a system of non-linear equations to hold the orthonormality of T as well as the zero coupling locations in M in terms of the unknowns in T. This system of non-linear equations can be solved numerically using the Newton-Raphson (NR) method, which converges quadratically near a possible solution point.

Starting with the system of non-linear equations

$$f_i(T_2, T_3, \cdots, T_{N-1}) = 0 \quad (2)$$

the linear Jacobian matrix J can be derived, where $i = 1,2,\cdots,\bar{n}$, $T_\#$ denotes the unknown # row of T and $\bar{n}$ is the total number of constraints of the system.

Since the first and last rows of T, and all elements of Λ are known, the following conclusions can be made.

- the number of unknowns L in T is given by

$$L = N(N-2) \quad (3)$$

- the number of constraints n_L to satisfy orthonormality of T can be written as

$$n_L = \sum_{i=3}^{N-1} i + N \quad (4)$$

- The elements M_{11}, M_{1N} (M_{N1}) and M_{NN} are constants and independent of unknown N-2 rows of T. Further $M_{1N}=0$ for a transmission coefficient having less than N-2 transmission zeros, which is the case for CT topology. Thus no constraint is required to set this coupling to zero.

With this information it is straightforward to find the maximum number of forced zero couplings. There are n_2 possible such zeros in the upper triangle of M (M is symmetrical), where n_2 is given by

$$n_2 = L - n_L \quad (5)$$

However, for some network topologies as the order increases the number of forced zero couplings becomes more than the maximum given in (5). In such cases, careful selection of the constraints could lead to a desired result. Section III now demonstrates how this procedure can be applied to synthesize a CT topology.

III. Application to Cascaded Triplet Topology

Fig. 1 shows the signal flow graphs of the cascaded triplets for N=5, 6 and 7.

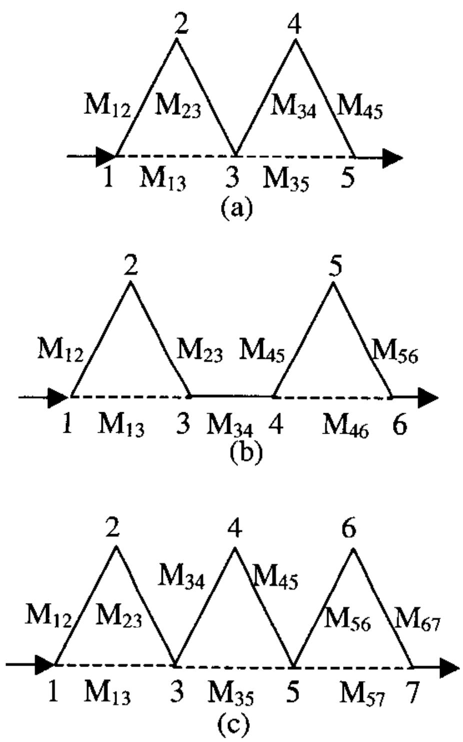

Fig. 1. Signal flow graphs for cascaded triplet topology (a) 5th order (b) 6th order (c) 7th order.

Table I provides the details to compose the system of non-linear equations to find the coupling matrix for the orders 5,6 and 7, using the proposed method. For a single triplet case ($N=3$) the unknown orthonormal row vector in T can be found by three constraints for orthonormality only. Thus the orthonormalization procedure is independent of the network topology. Therefore even the Gram-Schmidt procedure could provide the remaining row of T. However this is not the case for higher orders. For $N=5$, twelve constraints for orthonormality and three constraints $M_{14}=0$, $M_{24}=0$ and $M_{25}=0$ compose the system of equations to find the three remaining rows of T, using the proposed method.

TABLE I

Summary of Constraints Need to be Satisfied for Varying Filter Orders

N	L	n_L	Constraints for zero couplings
3	3	3	-
5	15	12	M_{14}, M_{24}, M_{25}
6	24	18	M_{24}, M_{25}, M_{35}, $(M_{14}, M_{15}, M_{26}, M_{36})$
7	35	25	$M_{24}, M_{25}, M_{26}, M_{36}$, M_{46}, M_{14}, M_{47}, $(M_{15}, M_{16}, M_{27}, M_{37})$

For orders 6 and 7, the number of constraints are one more than the degrees of freedom, making the system of equations unsolvable unless a redundancy is found. The following rules enable relaxation of one constraint, making the system of equations solvable by the proposed procedure.

1. Since the coupling matrix is real and symmetric no constraints for orthonormality can be relaxed.
2. Direct analysis of the numerator of transmission coefficient S_{21} calculated from the coupling matrix shows that it is possible to relax any one of the constraints to force a zero in the first row (M_{1k}) or last column (M_{kN}) of M, if either of M_{1k} or M_{kN} is non-zero in the final matrix, where $k = 2, 3, \ldots, N-1$. Note that other constraints cannot be relaxed in this way.

Thus any one of the constraints for zeros in M given in brackets can be relaxed for 6th and 7th orders. However this makes the system of equations to converge to two possible solutions depending on the initial values selected for the NR procedure. One solution leads to the CT realization, while the other contains a non-zero coupling value corresponding to the relaxed constraint. Using a priori knowledge of the two possible solutions, iterations can be repeated with a fresh set of initial values till it converges to the correct topology. The results given in the next section shows that it is possible to obtain a realizable coupling matrix following this procedure, with only a few iterations.

IV. Results

A 7th order asymmetric filter response with return loss 24 dB and transmission zeros at $\pm 1.5j, 2j$ is synthesized to demonstrate the validity of the proposed procedure. For the above specifications terminal resistances can be calculated as $R_1=1.1424\Omega$, $R_N=1.1423\Omega$. The proposed method extracts the following coupling matrix M for a CT topology

$$M=\begin{bmatrix} 0.0059 & -0.7825 & 0.4290 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.7825 & -0.5735 & -0.5079 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.4290 & -0.5079 & 0.0840 & 0.5060 & -0.2658 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.5060 & 0.5429 & 0.5028 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.2658 & 0.5028 & 0.0657 & 0.5645 & 0.2967 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5645 & -0.3988 & 0.8416 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2967 & 0.8416 & 0.0059 \end{bmatrix}$$

Here the constraint $M_{15}=0$ was dropped while forming the system of non-liner equations. The maximum residue of zero couplings of this resultant matrix is less than 10^{-12}. Fig. 2 shows the frequency response synthesized. The response calculated directly from the coupling matrix M coincides with this response, showing the accuracy of the proposed method.

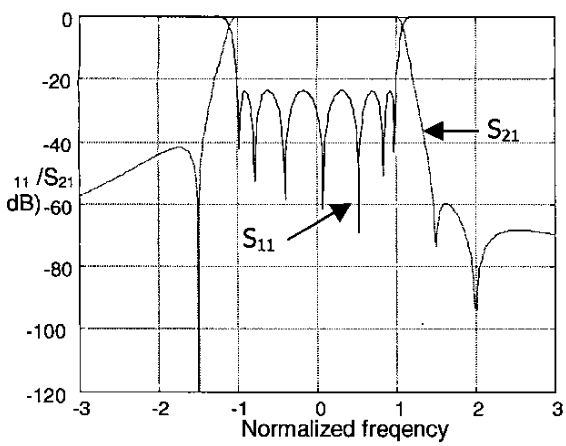

Fig.2. Transmission and reflection coefficient of filter response synthesized.

Table II provides the average number of iterations needed to obtain the final coupling matrix with a CT topology for orders considered in this paper. The average value is found by following the procedure with 20 different sets of initial values, which are uniformly distributed between 0 and 1. It can be observed that filter order 6 and 7 require more iterations, since the iteration procedure has to be repeated until the correct topology has been achieved. Note that the computation time needed for 100 iterations on a Pentium II, 450MHz PC is about 60 s for $N=7$.

TABLE II

Average Number of Iterations for Orders 5, 6 and 7 for a Return loss of 24 dB.

N	Locations of transmission zeros	Average no. of iterations
5	+/- 1.5j	11
	-1.5j, 3j	11
	1.5j, 3j	13
6	+/- 1.5j	57
	-1.5j, 2.4j	28
	1.5j, 2.4j	22
7	-1.5j, 1.8j, 2.2j	37
	-2j, 1.8j, 2.2j	38
	1.5j, 2.3j, 3j	92

V. Conclusions

A novel approach for the synthesis of cascaded triplets based on solving a non-linear equation set has been presented. As the order increases, it turns out that the number of equations become more than the degrees of freedom. However, it has been shown that careful selection of constraints makes it possible to obtain the required coupling matrix for orders 5, 6 and 7. Even though this method cannot be applied for higher orders due to the problem stated, the orders considered can provide the specifications required for many applications. Specially the sixth order topology could be used when cascading triple mode cavities to form a CT structure.

References

[1] A. E. Atia and A. E. Williams, "New type of waveguide bandpass filters for satellite transponders", *COMSAT Technical review*, vol. 1, pp. 21-43, Fall 1971.

[2] A. E. Atia and A. E. Williams, "Narrow-bandpass waveguide filters", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-20, pp. 258-265, April 1972.

[3] A. E. Atia and A. E. Williams, "Nonminimum-phase optimum-amplitude bandpass waveguide filters", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-22, pp. 425-431, April 1974.

[4] A. E. Atia, A. E. Williams and R.W. Newcomb, "Narrow-band multiple-coupled cavity synthesis", *IEEE Trans. Circuits and Systems*, vol. CAS-21, pp. 649-655, Sept. 1974.

[5] R. J. Cameron, "General coupling matrix synthesis methods for Chebyshev filtering functions", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-47, pp. 433-442, April 1999.

[6] G. Macchiarella, "Accurate synthesis of inline prototype filters using cascaded triplet and quadruplet sections", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-50, pp. 1779-1783, July 2002.